Neutrino Dark Energy – Revisiting the Stability Question

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Outline

1. Motivation
2. Neutrino Dark Energy–The Mass Varying Neutrino (MaVaN) Scenario
3. The Stability Issue
4. Summary

Neutrino Dark Energy (Mass Varying Neutrinos)

[Fardon,Nelson,Weiner '03]

Idea of varying neutrino masses in other contexts

[Kawasaki,Murayama,Yanagida '92,Stephenson et al '97]

- Attractive scalar force between Big Bang relic neutrinos (the analog of the Cosmic Microwave Background (CMB) photons)$\rightarrow$ smooth background, can form a negative pressure fluid
- $\rightarrow$ acts as a form of Dark Energy $\rightarrow$ accelerated expansion
- $\rightarrow$ neutrino mass m_ν becomes a function of neutrino energy density $\rho_\nu(z)$, which evolves on cosmological time scales (here parametrized in terms of cosmic redshift z)

$\rightarrow$ Neutrino mass not constant, but promoted to a dynamical quantity $m_\nu(z)$!

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Dynamics of the MaVaN Scenario

Complex interplay between the neutrinos and the scalar field

- Consider class of models $\mathcal{L} \supset \mathcal{L}_\phi + \mathcal{L}_{\nu_{\rm kin}} + \underbrace{\mathcal{L}_{\nu_{\rm mass}}}_{-m_\nu(\phi)\bar{\nu}\nu}$
- $\rightarrow m_\nu(\phi)$ generated by light scalar field, $\phi \rightarrow$ become linked to its dynamics
- Neutrino energy density ρ_ν and pressure p_ν are functions of neutrino mass $m_\nu(\phi)$ $\longrightarrow \rho_\nu(m_\nu(\phi)), p_\nu(m_\nu(\phi))$
- $\longrightarrow$ Neutrinos can stabilize ϕ by contributing to its effective potential $V_{\rm eff}(\phi) = [\rho_\nu(m_\nu(\phi)) - 3p_\nu(m_\nu(\phi))] + V_\phi(\phi)$
- Evolution of ϕ governed by modified Klein-Gordon equation

with a denoting the scale factor and $H \equiv \frac{\dot{a}}{a}$ the Hubble expansion rate

$$\ddot{\phi} + 2H\dot{\phi} + a^2 V_\phi' = -a^2 \underbrace{\frac{d\log m_\nu}{d\phi}}_{\text{coupling } \beta}(\rho_\nu - 3p_\nu), \quad \text{with } (' = d/d\phi)$$

- Extra source term on RHS accounts for energy exchange of ϕ and neutrinos
- As long as neutrinos relativistic, coupling term suppressed $(\rho_\nu - 3p_\nu \sim 0)$

Dynamics of the MaVaN Scenario

Adiabatic evolution in the non-relativistic neutrino regime

- Consider late-time dynamics of MaVaNs in the non-relativistic limit $m_\nu \gg T_\nu$ $\rightarrow p_\nu \sim 0,\ \rho_\nu = m_\nu n_\nu$ ($n_\nu \equiv$ neutrino number density)

  $\longrightarrow V_{\rm eff}(\phi) = \rho_\nu(m_\nu(\phi)) + V_\phi(\phi)$

- Due to stabilizing effect of neutrinos on ϕ, model can accomplish late-time acceleration also for $m_{\phi,0} \gg H_0 \sim 10^{-33}\mathrm{eV}$
- In the limit $m_\phi^2 \equiv V''_{\rm eff}(\phi) \gg H^2$ adiabatic solution to EOM of ϕ apply (Recall EOM: $\ddot\phi + 2H\dot\phi + a^2 V'_{\rm eff}(\phi, z) = 0$, can safely neglect effects of kinetic energy terms)
- $\longrightarrow \phi$ instantaneously tracks the minimum of its effective potential $V_{\rm eff} \rightarrow$

$$V'_{\rm eff}(\phi, z) = V'_\phi(\phi) + \underbrace{\rho'_\nu(m_\nu(\phi), z)}_{m'_\nu(\phi) n_\nu(m_\nu(\phi), z)} = 0 \text{ with } (' = \partial/\partial\phi)$$

Crucial effect: $\rho_\nu(m_\nu(\phi), z)$ is diluted by expansion $\rightarrow \phi$ varies on cosmological time scales (slowly)

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Neutrino mass varies!

- $m_\nu(\phi) = m_\nu(\phi, z)$, $\rightarrow$

  $V_{\text{eff}}(\phi, z) = V_{\text{eff}}(m_\nu(\phi), z)$

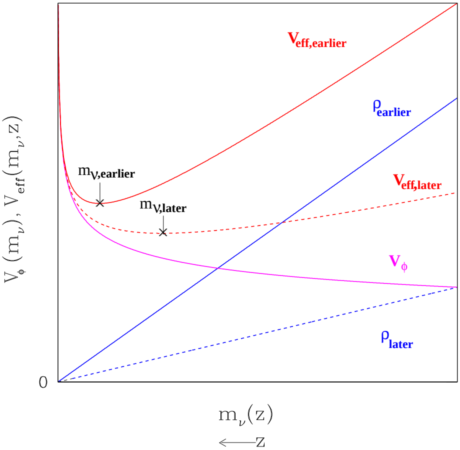
  $= \underbrace{\rho_\nu(m_\nu(\phi), z)}_{m_\nu(\phi) n_\nu(z)} + V_\phi(m_\nu(\phi))$

- $\rightarrow \frac{\partial V_{\text{eff}}(\phi)}{\partial \phi} =$
  $\frac{\partial m_\nu}{\partial \phi} \frac{\partial V_{\text{eff}}(m_\nu)}{\partial m_\nu} |_{m_\nu = m_\nu(\phi)} = 0$

- Neutrino mass variation determined from $\frac{\partial V_{\text{eff}}(m_\nu, z)}{\partial m_\nu} =$
  $0 = n_\nu(m_\nu, z) + \frac{\partial V_\phi(m_\nu)}{\partial m_\nu}$

$\rightarrow$ Combined scalar-neutrino fluid has dynamical Eq. of State $\omega(z) \equiv \frac{p_{\text{DE}}(z)}{\rho_{\text{DE}}(z)}$

$$\omega(z) + 1 = -\frac{m_\nu(z) V'_\phi(m_\nu(z))}{m'_\nu(z) V_{\text{eff}}(m_\nu(z))}$$



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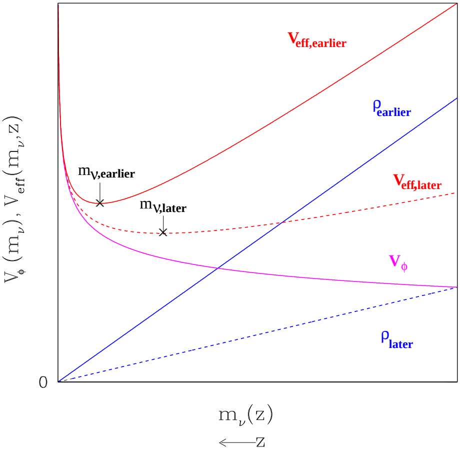

The Stability Issue in Models of Neutrino Dark Energy

Instabilities? Formation of dense neutrino bound states?

'In the non-relativistic neutrino regime any realistic MaVaN scenario with $m_\phi^2 \gg H^2 > 0$ is characterized by a negative sound speed squared $c_s^2 < 0$ and thus becomes unstable to hydrodynamic perturbations...with the likely outcome of the formation of non-linear structures in the neutrino density ('neutrino nuggets')'

[Afshordi, Kohri, Zaldarriaga '05]

Note: Outcome of neutrino instability is an inherently non-linear process ...but if 'nuggets' really form, they redshifts similar to cold dark matter with $\omega \sim 0 \not\approx -1 \rightarrow$ no acceleration (Quintessence? Cosmological Constant?)

Reconsider stability issue in framework of linear perturbation theory

Aim: Identification of condition for stabilization of the neutrino density contrast

Instabilities

- Neutrino instabilities driven by attractive force mediated by ϕ
- Phenomenon similar to gravitational instabilities of CDM
- Good observational evidence, at early times universe homogeneouse and isotropic on all scales
- Apart from small primeval peturbations $\delta\rho_i$ in densities ρ_i of each individual particle i

$$\rho_i(\mathbf{x},\tau) = \underbrace{\rho_i(\tau)}_{\text{mean background density}} + \underbrace{\delta\rho_i(\mathbf{x},\tau)}_{\text{small perturbation}} \quad , \qquad \underbrace{\delta_i(\mathbf{x},\tau) \equiv \frac{\delta\rho_i(\mathbf{x},\tau)}{\rho_i(\tau)}}_{\text{density constrast}}$$

- $\rightarrow$ grew by gravity into observable structure on scales of galaxies and clusters of galaxies
- Small amplitudes $|\delta\rho_i(\mathbf{x},\tau)| \ll \rho_i(\tau) \leftrightarrow |\delta_i(\mathbf{x},\tau)| \ll 1 \rightarrow$ growth of fluctuations can be solved from linear perturbation theory

The Stability Issue in Models of Neutrino Dark Energy

Gravitational instability in Newtonian theory

- Assume static (non-expanding) universe, consider perfect fluid, density ρ, pressure p, velocity $\mathbf{v}$ (Continuity eq. + Euler eq. + Newtonian gravity)
- Add small perturbations δp, $\delta\rho$, $\delta\mathbf{v}$ and linearise $\rightarrow$ for k^{th} Fourier component

$$\ddot{\delta}_k + (\underbrace{c_s^2 k^2}_{\text{pressure}} - \underbrace{4\pi G\rho}_{\text{gravity}})\,\delta_k = 0, \quad \text{where} \quad \omega = \sqrt{c_s^2 k^2 - 4\pi G\rho}$$

- Perturbations adiabatic ($c_s^2 = \frac{\dot{p}}{\dot{\rho}}$ adiabatic sound speed squared)
- $\rightarrow$ sign of ω^2 (which depends on c_s^2) determines perturbation evolution
- change of sign of ω^2 at critical value $k_{\text{Jeans}} = \sqrt{4\pi G\rho/c_s^2}$
- for $k < k_{\text{Jeans}}$: $\omega^2 < 0$ (gravity overcomes pressure) $\rightarrow$ $\delta_k \propto e^{\pm|\omega|t}$, growing solution
- for $k > k_{\text{Jeans}}$: $\omega^2 > 0 \rightarrow$ $\delta_k \propto e^{\pm i\omega t}$, no growth but acoustic oscillations

$\rightarrow$ sound speed squared c_s^2 governs evolution of density contrast δ_k

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Make contact with MaVaN instabilities

- MaVaNs interact through gravity and the force mediated by ϕ (both attractive), $4\pi G \rightarrow 4\pi G_{\rm eff}(\beta(\phi))$
- Sound speed squared? For a general fluid i (with 'c_g' general, 'c_S' adiabatic, 'Γ_i' intrinsic entropy perturbation)

$$w_i \Gamma_i = (c_{gi}^2 - c_{ai}^2)\, \delta_i, \qquad c_g^2 = \frac{\delta p_i}{\delta \rho_i}, \qquad c_S^2 = \frac{\dot{p}_i}{\dot{\rho}_i}, \qquad \delta_i(\mathbf{x}, \tau) \equiv \frac{\delta \rho_i(\mathbf{x}, \tau)}{\rho_i(\tau)}$$

- Dissipative processes invoke entropy perturbations ($\Gamma_i \neq 0$) [Hu' 98, Bean, Dore '03, ...]
- For MaVaNs? Depends on scales/regimes one considers!
- Relativistic neutrinos: free-streaming and relativistic pressure support $\rightarrow$ no growth (on all scales)
- Non-relativistic neutrinos: $p_\nu \sim 0 \rightarrow$ possible growth
- m_ϕ^{-1} sets physical length scales a/k as of which gradient terms become unimportant ($\Gamma_\phi \sim 0$) (for small deviations away from its minimum, ϕ re-adjusts to new minimum on a time scale $m_\phi^{-1} \ll H^{-1}$)

[Afshordi, Kohri, Zaldarriaga ' 05, Kaplinghat, Rajaraman' 06]

On scales $m_\phi^{-1} < a/k < H^{-1}$ MaVaN perturbations adiabatic $\rightarrow \nu - \phi$ system can be treated as unified fluid with $\Gamma_{DE} = 0$ and $c_S^2 = \frac{\dot{p}_{\rm DE}}{\dot{\rho}_{\rm DE}} = \omega - \frac{\dot{\omega}}{3H(1+\omega)}$

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Equation of motion of the neutrino density contrast $\delta_\nu = \frac{\delta\rho_\nu}{\rho_\nu}$

- Energy-momentum conservation equations for the coupled neutrinos

$$T^{\mu}_{\gamma;\mu} = \underbrace{\frac{d \log m_\nu}{d\phi}}_{\text{coupling } \beta} \phi_{,\gamma} T^{\alpha}_{\alpha},$$

where $T_{\mu\gamma}$ is the energy-momentum tensor

- consider perturbed part in the non-relativistic neutrino regime

(where instabilities can possibly grow)

- use perturbed part of the Klein-Gordon eq. $\ddot{\delta\phi} + 2H\dot{\delta\phi} + [k^2 + a^2(V''_\phi + \beta'\rho_\nu)]\delta\phi = -a^2\beta\delta_\nu\rho_\nu$

[cf. eg. Amendola '03, Koivisto '05]

$$\delta\phi = -\frac{a^2\beta\rho_\nu\delta_\nu}{a^2(V''_\phi + \beta'\rho_\nu) + k^2}$$

Equation of motion of the neutrino density contrast $\delta_\nu = \frac{\delta\rho_\nu}{\rho_\nu}$

In the non-relativistic neutrino regime on length scales $m_\phi^{-1} < a/k < H^{-1}$ with negligible neutrino shear and $p_\nu \sim \omega_\nu \sim 0$

Compare: Newtonian theory, static universe, perfect fluid $\ddot{\delta} + (c_s^2 k^2 - 4\pi G\rho)\delta = 0$

$\delta_\mathrm{b} \simeq \delta_\mathrm{CDM}$ deep in matter-dominated regime

$$\ddot{\delta}_\nu + H\dot{\delta}_\nu + [c_\nu^2 k^2 - 4\pi a^2 G_\mathrm{eff}\rho_\nu]\delta_\nu = 4\pi a^2 G\,[\rho_\mathrm{CDM}\delta_\mathrm{CDM} + \rho_b\delta_b]$$

$$G_\mathrm{eff} = G\left[1 + \frac{2\beta^2 M_\mathrm{pl}^2}{1+a^2(V_\phi'' + \beta'\rho_\nu)/k^2}\right]$$

$$G\,[1 + 2\beta^2 M_{pl}^2] \gtrsim G_\mathrm{eff} \gtrsim G$$

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deep in matter-dominated regime

$$\ddot{\delta}_\nu + H\dot{\delta}_\nu + \overbrace{[c_\nu^2 k^2 - \frac{3}{2}H^2\frac{G_{\rm eff}}{G}\underbrace{\Omega_\nu}_{\sim 10^{-4}..0.02}]}^{\simeq(\frac{c_s^2}{(c_s^2+1)}k^2 - \frac{3}{2}H^2\Omega_\nu)\delta_\nu}\delta_\nu = \frac{3}{2}H^2[\underbrace{\Omega_{\rm CDM}}_{\sim 0.22} + \underbrace{\Omega_b}_{\sim 0.04}]\delta_{\rm CDM}$$

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Dynamics of δ_ν governed by CDM $\rightarrow$ moderate growth like ordinary gravitational instabilities ('neutrinos follow CDM') $\rightarrow$ up to the present time $\delta_\nu \ll 1$ ($\equiv$ stability) possible	

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Any realistic MaVaN scenario $c_s^2 < 0$?

- Require $c_s^2 = \frac{\dot{p}_{\mathrm{DE}}}{\dot{\rho}_{\mathrm{DE}}} = \omega - \frac{\dot{\omega}}{3H(1+\omega)} \geq 0$ for $m_\nu(z) \gg T_\nu(z)$ (take into account finite temperature effects)

$$\rightarrow \sum_{i=1}^{3} \frac{\partial m_{\nu_i}(z)}{\partial z} \left(1 - \frac{5\alpha T_{\nu,0}^2 (1+z)^2}{3m_{\nu_i}^2(z)}\right) + \sum_{i=1}^{3} \frac{25\alpha T_{\nu,0}^2(1+z)}{3m_{\nu_i}(z)} \geq 0, \text{ with } \alpha \equiv \frac{\int_0^\infty \frac{dy\, y^4}{e^y+1}}{2\int_0^\infty \frac{dy\, y^2}{e^y+1}}$$

[Takahashi,Tanimoto '06]

- Assume degenerate mass spectrum with $m_{\nu_i}(0) \sim m_\nu(0) = 0.312$ eV, $i = 1, 2, 3$ $\rightarrow$ determine maximally allowed neutrino mass variation

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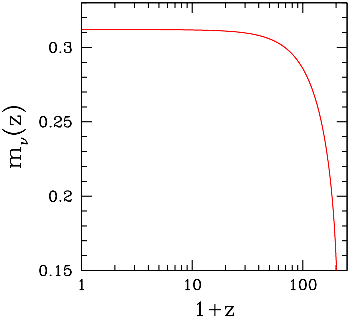

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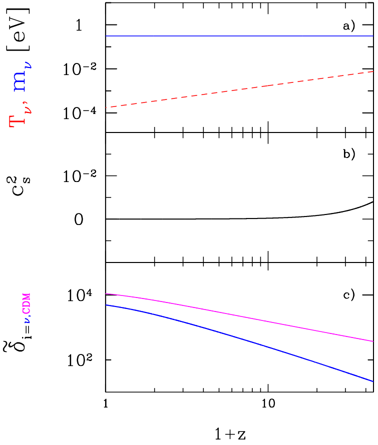
  $\rightarrow$ determine maximally allowed neutrino mass variation

Requirement of $c_s^2 \geq 0$ strongly restricts the allowed mass variation at late times

A stable model

- Normalization?
- $\rightarrow$ For $k = 0.11h\,\mathrm{Mpc}^{-1}$
  $\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2} \propto \delta^2_{\mathrm{CDM}} \ll 1 \rightarrow$ linear

  [Percival et al.'06]
- Since $\tilde{\delta}^2_\nu < \tilde{\delta}^2_{\mathrm{CDM}} \rightarrow$ neutrino density contrast linear, $\delta^2_\nu \ll 1$ = no 'neutrino nuggets'!
- $\rightarrow$ Adiabatic model of Neutrino Dark Energy stable also in the highly non-relativistic regime$\rightarrow$ viable dark energy candidate



$k = 0.11h\mathrm{Mpc}^{-1}$, $\beta = 1/M_{\mathrm{pl}}$, $m_{\nu_i}(z = 0) = 0.312$ eV

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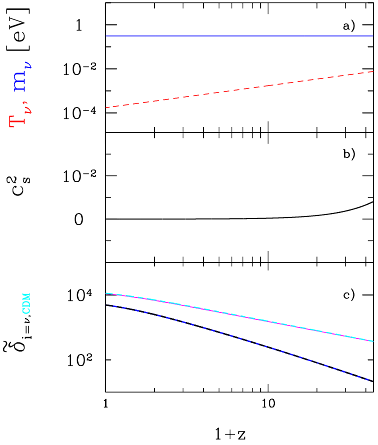


$k = 0.11h\mathrm{Mpc}^{-1}$, $\beta = 0$, $m_{\nu_i}(z=0) = 0.312$ eV

The Stability Issue in Models of Neutrino Dark Energy

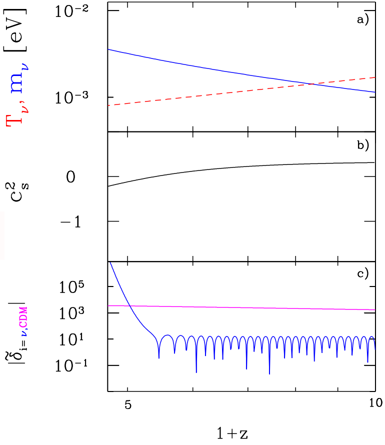
An unstable model

- Rapid evolution of $m_\nu(z)$
- $m_\nu(z) \ll T_\nu(z)$ (non-relativistic regime): pressure support deminishes $\rightarrow$ c_s^2 driven to negative values

Recall:

$$\ddot{\delta}_\nu + H\dot{\delta}_\nu + [c_\nu^2 k^2 - \frac{3}{2}H^2 \frac{G_{\rm eff}}{G}\Omega_\nu]\delta_\nu = \frac{3}{2}H^2 \, [\Omega_{\rm CDM} + \Omega_b] \, \delta_{\rm CDM}$$

- As soon as coupling is large enough to compensate for small neutrino mass (and thus Ω_ν) $\rightarrow$
- $\delta_\nu \gg 1$ $\rightarrow$ model unstable before today $\rightarrow$ excluded as DE candidate



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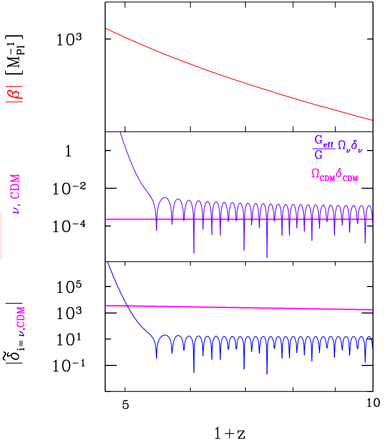

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Summary

- Reconsideration of the stability issue in models of adiabatic neutrino dark energy
- Other cosmic components (CDM and baryons) can have stabilizing effect on MaVaN perturbations
- If $\frac{G_{\text{eff}}}{G}\Omega_\nu \ll [\Omega_{\text{CDM}} + \Omega_{\text{b}}] \rightarrow$ moderate growth of perturbations as in general relativity $\rightarrow \delta_\nu \ll 1$ ($\equiv$ stability) possible
- If strong coupling compensates for relative smallness of $\Omega_\nu \rightarrow \delta_\nu \gg 1$ ($\equiv$ instability) in the non-relativistic regime
- Viable model of neutrino dark energy found with $c_s^2 > 0 \rightarrow$ allowed mass variation strongly restricted at late times
- Note: non-adiabatic models of neutrino dark energy with $m_\phi \sim H$ are stable [Brookfield, van de Bruck, Mota, Tocchini-Valentini '06, Afshordi, Kohri, Zaldarriaga '05]
- Note: 'Hybrid' models involving two light scalar fields can be stable until the present time even in the presence of unstable neutrino component

[Fardon, Nelson, Weiner '06, Spitzer '06]

Appendix

Mass Varying Neutrino (MaVaN) Scenario

The non-SM neutrino interaction mediated by a scalar field

- Introduce a light scalar field ϕ with mass $H_0 \sim 10^{-33}\mathrm{eV} \ll m_\phi \lesssim 10^{-4}\mathrm{eV}$
- Introduce a coupling between neutrinos ν and ϕ
- $\rightarrow$ Consider class of models with

$$\mathcal{L} \supset \mathcal{L}_\phi + \mathcal{L}_{\nu_{\mathrm{kin}}} + \mathcal{L}_{\nu_{\mathrm{mass}}}, \text{ where}$$

$$\mathcal{L}_\phi = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V_\phi(\phi)$$

$$\mathcal{L}_{\nu_{\mathrm{mass}}} = -m_\nu(\phi)\bar{\nu}\nu$$

$\rightarrow$ neutrino mass $m_\nu(\phi)$ is generated from the VEV of ϕ and becomes linked to its dynamics
$\rightarrow$ neutrinos interact through a new non-SM force

Evolution of scalar field perturbations $\delta\phi$

where $\phi(\mathbf{x},\tau) = \phi(\tau) + \delta\phi(\mathbf{x},\tau)$

- Perturbed Klein-Gordon equation in the non-relativistic neutrino regime ($\rightarrow$ neglect terms $\propto p_\nu, \omega_\nu, c_\nu^2$ and $\dot\phi$)

$$\ddot{\delta\phi} + 2H\dot{\delta\phi} + \left[k^2 + a^2 \underbrace{\left(V''_\phi + \beta'\rho_\nu\right)}_{m_\phi^2 - \beta^2\rho_\nu}\right]\delta\phi = -a^2\beta\delta_\nu\rho_\nu$$

- Solution of homogenous equation is oscillating with decaying amplitude
- Particular solution given by forcing term on RHS

$$\delta\phi = -\frac{a^2\beta\rho_\nu\delta_\nu}{a^2(V''_\phi + \beta'\rho_\nu) + k^2}$$

[cf. eg. Amendola' 03, Koivisto' 05]

A concrete model

Consider model proposed in the context of 'Chameleon cosmologies' [Khoury, Weltman '03, Brax, van de Bruck, Davis, Khoury, Weltman '04, ...]

- Recall: evolution of ϕ determined by $V'_{\rm eff}(\phi) = 0 = V'_\phi(\phi) + \rho'_\nu(m_\nu(\phi))$
- Exponential potential

$$V_\phi(\phi) = M^4 e^{\frac{M^n}{\phi^n}}$$

- Exponential dependence of m_ν on ϕ

$$m_\nu(\phi) = m_0 e^{\beta\phi}, \text{ where } \overbrace{\beta}^{\text{coupling}} = \frac{d \log m_\nu}{d\phi} = \text{const.}$$

- Typically, $\beta\phi \ll 1 \rightarrow m_\nu$ very weakly depends on changes in the neutrino energy density $\rightarrow$ m_ν hardly evolves with time
- $\rightarrow$ attractive force between neutrinos essentially time independent

Another model

Proposed by Fardon, Nelson, Weiner '05

- Logarithmic scalar potential ($V0$ fixed by requirement of $\Omega_{\rm DE} \sim 0.7$)

$$V_\phi(\phi) = V_0 \log(1+\kappa\phi), \text{ with } V_0, \kappa = \text{const.}$$

- Mass dependence on ϕ as preferred in the MaVaN literature [Fardon, Nelson, Weiner '05, '06, Afshordi, Kohri, Zaldarriaga ' 05, Spitzer' 06...]

$$m_\nu(\phi) = \frac{m_0}{\phi}, \text{ where } \overbrace{\beta}^{\text{coupling}} = \frac{d\log m_\nu}{d\phi} = -\frac{1}{\phi} \neq \text{const.}$$

- dependence $m_\nu(\phi)$ naturally arises from integrating out a heavier sterile state, whose mass varies linearly with the value of ϕ ('MaVaN seesaw')
- m_ν strongly depends on changes in the scalar field VEV
- since ϕ decreases, $|\beta|$ increases with time $\rightarrow$ attractive force between neutrinos increases with time